
Warden Point, Isle of Sheppey, Kent

[TR 018 726]

Introduction

The cliffs in London Clay at Warden Point exhibit a series of deep-seated rotational slips (Figure 8.5) which are bench-shaped in plan, and generally extend along the coast for distances of between 3.1 and 7.1 times the cliff height, and average in width 5.0 times the cliff height (Figure 8.6).

Warden Point is the most exposed northeastern extremity of the cliffs of the Isle of Sheppey, Kent, facing the Thames estuary (see (Figure 8.6)). It is formed in cliffs of London Clay (Eocene) (Figure 8.7) up to 40 m high, which stretch from Warden Point for 8 km to the east). According to Steers (1964), marine erosion is generally strong along the length of the cliffs giving a maximum recession rate of nearly 3 m a⁻¹ at their more exposed eastern end. Over the period 1865–1964 rates vary from 1.5 m a⁻¹ to 2.15 m a⁻¹ (Hutchinson, 1973).

Description

Throughout the last 100 years, the average length of time between consecutive first-time slides occurring at a particular length of the cliff has been 30–40 years. Hutchinson (1965), in conducting a survey of all the landslides on the Kent coast, labelled the landslides around Warden Point as K37 to K40; the positions of these are shown in (Figure 8.6). Hutchinson intended these labels to refer to the individual slide masses, but, following Dixon and Bromhead (1991), they are used here to denote the sections (lengths) of cliff in which the slides occur.

(a) Section K40

A first-time slide took place at K40 during the night of Friday 13 January 1989. It involved a length of cliff-top about 150 m long, with a maximum land loss of 25–30 m. The landslide took the form of a large back-tilted block of relatively intact London Clay, which moved about 12 m down the cliff during the initial failure. The original cliff-top ground surface, recognizable by the grassed and partly wooded back-tilted area, remained predominantly undisturbed by internal shearing. However, immediately seaward of the original cliff edge a secondary scarp indicated some internal disturbance. The toe of the slide experienced 3 m of heave as a result of rotation of the slide mass. This rotation was considerable, and the absence of excessive internal distortion of the slide mass indicates that the slip-surface was close to being circular in shape (Dixon and Bromhead, 1991).

The previous slide at K40, which occurred in 1950, also involved a back-tilted slide mass with little associated disturbance (Hutchinson, 1965). In front of this length of cliff the foreshore comprises the planed-off remnants of previous slides. These can be recognized by linear outcrops of septarian nodules (formed by originally sub-horizontal nodule beds having been rotated during the slide movements), and by the end shears, which clearly mark the boundary between the landslipped material and the in-situ clay. The in-situ clay marks the position of the former abutments between laterally adjacent slides. There are no lines of nodules cropping out in these areas of in-situ material, indicating that the slides at this location have been repeatedly deep-seated (Dixon and Bromhead, 1991).

(b) Section K39

The most recent failure in the K39 section of the cliff took place in November 1971, and was described by Hutchinson (1971). The initial slide movements involved a 200 m length of cliff-top with a maximum width of 30 m at its centre. From the evidence on the foreshore the slides at this location are also deep-seated.

Post-failure observations at K39 indicate that the shape of the slip-surface varies across the length of the slide (Dixon and Bromhead, 1991). Along the eastern and central section the landslide mass moved 18 m down the slope, back-tilted

by 16°, and remained relatively intact. This suggests that at least the rear section of the slip-surface was close to being circular in shape. In the western part of the slide mass, which had an initial downward displacement of 15 m, not only was there a 7° angle of back-tilt, but also the landslide block was severely distorted by internal shearing; this included the formation of a graben-type feature with reverse shears and a zone of tension cracks which were up to 10 m deep. Thus it seems that in the western section of the cliff, failure took place involving a highly non-circular slip-surface (Dixon and Bromhead, 1991).

(Figure 8.8) a shows a cross-section through the central part of the slide mass which was surveyed by Gostelow (1974) shortly after the first-time failure. Also shown is the slope profile in 1985 and the position of the slip-surface as indicated by the above measurements and observations. The measured slip-surface is consistent with the observed mode of failure (Dixon and Bromhead, 1991).

(c) Section K38

A rotational slide occurred at K38 in 1945. A local resident who remembered the slide happening reported that a strip of cliff-top about 30 m wide was involved in the initial failure (Hutchinson, 1965). In addition, the slipped mass is reported to have exhibited considerable back-tilt. In front of this length of cliff the foreshore is formed of in-situ London Clay, and therefore it can be assumed that the slip-surfaces do not extend below about 0 m OD.

The section of cliff at K38 commenced falling around May 1988 while instrumentation was in place; deformations continued over a period of about 6 weeks. A detailed preliminary study was carried out by Koor (1989) who surveyed and mapped the slide area: it was clear that the failure was not a rotational slide. The following description and assessment of the failure mode are taken from his work.

Failure was initiated at the north-west end of the section and spread over a six-week period south-eastwards along the cliff. There are two distinct areas of landslipped material that are separated by a fault in the London Clay. The fault has a throw of about 1 m and should therefore be at residual shear-strength. The existence of the fault clearly controlled the two-phase nature of the movements. A cross-section surveyed through the eastern part of the slipped mass is shown in (Figure 8.8)b.

The following features were associated with the failure:

1. a zone of tension cracks extending up to 30 m landward of the main scarp, with associated settlement of the cliff crest;
2. the main slide mass was formed from forward rotating linear blocks of London Clay which had slickenslided counterscarps;
3. the toe of the slope was formed of scree-like material.

Koor (1989) proposed failure by flexural toppling as an explanation of the above features. This type of failure could occur as a result of vertical columns of London Clay being formed by stress-relief-induced vertical discontinuities. If these columns were not adequately supported by the toe of the slope, they would tend to topple seawards thus producing the slope profile observed. This failure mode would account for the retrogressive nature of the slope movements. Also shown in (Figure 8.8)b is the pre-failure slope profile. By comparing these two sections it is evident that substantial translational deformations took place.

(d) Section K37

Two major rotational slides occurred at K37 during the 1970s. In 1973 a slip took place in the western half of the cliff and in 1974 the remaining length failed. Although both landslide blocks exhibited a considerable amount of back-tilt, this was accompanied by major internal shearing, as demonstrated by the presence of ridges on the surface of the slide mass and a large number of deep tension cracks. From this evidence it is clear

that the slip-surface was non-circular. Inspection (4) of the foreshore shows that previous slides at this location were not deep-seated. (Figure 8.8)c shows a cross-section through the 1974 landslide mass surveyed in 1975 by Lee (1976).

At the eastern end of K37 the base of the slip-surface outcrops in the cliff which forms the toe of the slope. The slip-surface follows the dip of the bedding, which is indicated by layers of septarian nodules exposed in the cliff-face, and is formed of approximately 50 mm of highly sheared re-moulded clay with the primary shear (⁵) surface positioned at the base of the failure zone.

Interpretation

(a) Hutchinson

Hutchinson (1968b, 1973) described a form of cyclic mechanism of mass movement in London Clay cliffs. He illustrated this mechanism using examples from a range of sites as well as Warden Point, including The Lees and Beltinge at Herne Bay, and compared them with the mechanisms that operate on inland slopes. The stages are as follows (Figure 8.9):

1. A deep-seated landslide is imminent. The virtually stable upper cliff is blanketed by thin deposits of exhausted mudslides, commonly grassed-over at inclinations of about 15°–20°. The lower cliff is bare, in-situ clay under active marine attack.
2. The landslide has just taken place. It is characteristically a deep-seated rotational slip involving base failure. In failing, the slipped mass has moved downwards and seawards, rotating backwards in the process, until a position of equilibrium is again reached, commonly when the former cliff edge is roughly halfway down the cliff. Pools of water frequently collect in the hollow thus formed.
3. This stage comprises two main processes: the commencement of marine erosion at the toe of the slipped mass and the rapid breakdown of the steep rear scarp of the slip, chiefly by soil-fall and shallow slides. The debris from these collects in the hollow at the back of the slipped mass. This loading, combined with the unloading resulting from the toe erosion, brings about a slight further rotation and sinking of the landslide.
4. Erosion at the landslide toe has reached the line of the cliff-foot just before the slip occurred. The continuing degradation of the rear scarp is now affected predominantly by mudslides, chiefly during the wet season. The deposits of these gradually fill the hollow behind the slipped mass and begin to spill into the sea to form secondary mudslides around each extremity of the slipped mass and further slight rotation and sinking of the mass results.
5. The gradual removal of the toe of the slipped mass, and the degradation of the rear scarp, continues. The lower cliff is now completely buried under mudslide deposits. The mudslides on the upper cliff are approaching a state of exhaustion, having almost destroyed the rear scarp from which their debris supply largely derived, and degraded to an inclination too low for an effective rate of debris transport to be maintained. The two secondary mudflows, however, being continuously stimulated by marine erosion, increase in power and enlarge their channels at the expense of the slipped mass and the sides of the landslide cavity. The sea continues to erode the face of the slipped mass and may cause it to sag forward and so reduce its degree of back-tilt. Vertical erosion of the slipped mass forming the foreshore occurs.

In the following stage, the sea, having removed the last remains of the slipped mass remains above sea-level, once again attacks the in-situ clay. The cycle has been completed and the cliff, displaced somewhat inland, again stands at stage 1 on the point of a deep-seated failure.

The time taken to complete this cycle appears to vary primarily with the intensity of the erosion and to a lesser extent with cliff height. At Warden Point the cycle length is estimated to have been 30–40 years (Hutchinson, 1973). A cliff undergoing this type of cyclic degradation exhibits, not parallel retreat, but a mode of retreat which alternates in inclination between the steep-angled stage 1 and the shallow-angled stage 4. This results from the alternation of the predominantly erosional phase of cliff steepening with the succeeding phase dominated by degradation, initially by deep-seated landslide and subsequently by shallow slides and mudslides.

In plan view (see (Figure 8.6)) it is clear that the slips take place along the same axis and that the headlands between them maintain their positions. It is likely that this is due to the low porewater-pressure and because the debris is removed at the foot. This is in contrast to the landslides at Ventnor, Isle of Wight, where lack of removal at the foot of the slope results in offsetting of headlands in successive cycles of sliding.

(b) Bromhead and Dixon on depressed porewater-pressure and its implications

Bromhead and Dixon have adopted two main lines of investigation of the cliffs at Warden Point. Firstly, they have investigated porewater-pressure distribution in detail (Bromhead and Dixon, 1984; Dixon and Bromhead, 1986). This was done not so much to investigate the site's characteristics, as to use the site as a testbed for the idea that a condition often observed in unloaded artificial slopes (most commonly at excavations) might also exist in certain natural slopes. The condition is that stress-relief on excavation leads to reduced porewater-pressures around the excavation ((Figure 8.10); Vaughan and Walbancke, 1973; Eigenbrod, 1975). Skempton (1977) attributed delayed failure of cutting slopes in brown London Clay to this effect. According to Dixon and Bromhead (1986), it can be expected that the effect will be modified in the case of a natural slope, by a mantle of shallow landslide deposits (e.g. mudslides; Hutchinson, 1967), with a localized, perched, and possibly artesian porewater-pressure regime (Hutchinson and Bhandari, 1971). If, in addition, the slope has been subject to deep-seated slide activity; it may be grossly modified by porewater-pressure distributions 'carried down' the slope, but otherwise unaltered (Dixon and Bromhead, 1986).

To measure the magnitude and distribution of a depressed porewater-pressure regime in a slope, the following criteria must be met:

1. a large slope is needed to enable an adequate number of measurements to be made;
2. the slope must be subject to rapid erosion so that the porewater-pressure changes are recent and not partly equilibrated (this is important owing to uncertainty about field equilibration rates (Chandler, 1984));
3. the magnitude of porewater-pressure depression should be great so that inevitable inaccuracies in measurements are immaterial.

These criteria are met by the 46 m-high London Clay cliffs at Warden Point. Two sections were instrumented by Bromhead and Dixon (1984, 1986; Dixon and Bromhead, 1986) to allow differentiation between porewater-pressure changes resulting from stress-relief, and those 'carried down' the cliff when one of the characteristic large and deep-seated rotational slides takes place.

Dixon and Bromhead (1986; Bromhead and Dixon, 1984) installed 56 piezometers along two cross-sections (Figure 8.10)a,b, which extend back behind the cliff-top, and onto the foreshore. The piezometric levels were read at about 10-day intervals. The two sections were chosen with the intention that Section 1 (Figure 8.10)a would yield important data on the initial failure of the cliff, providing peak field strength values from back-analysis. Section 2 (Figure 8.10)b would yield data for an existing landslide and provide a field residual strength. Each piezometer was installed in a PVC plastic tube located in a vertically augered hole (maximum available depth 45 m). Blockages in the tubing due to shearing or bending were used to identify the position of zones of movement. Most of the piezometers were far too deep for seasonal effects to be significant, and in fact no seasonal effects were recorded.

Dixon and Bromhead (1986) did not find it possible to explain the measured porewater-pressure distribution in terms of a steady seepage regime, which is the usual replenishment mechanism. At Warden Point the cliffs are composed of the uppermost part of the London Clay, which is 132 m thick at this location. The London Clay extends to a depth of 83 m below beach level at Warden Point, with a small amount of underdrainage taking place into the underlying sands of the Oldhaven Beds. This base boundary is not critical, the slope being isolated from the underdrainage by decreasing permeability with depth (Bromhead and Vaughan, 1980). The measured porewater pressures are consistently less than those predicted by seepage analyses.

Taking into consideration the depressed porewater-pressures found in artificial cutting slopes as discussed above, it seems likely that, as in such slopes, the low porewater-pressures at Warden Point are due to stress-relief effects. The difference between the measured and predicted porewater-pressures decreases as the distance inland from the cliff

edge increases. It appears that the zone of depressed porewater-pressure is carried inland with the cliff as the coastline retreats. It is local to the slope, but may extend a comparatively large distance inland. The predicted equilibrium levels shown in (Figure 8.10) were derived by Bromhead and Dixon (1984) from water-level/time plots using Gibson's (1963) theory and a curve-fitting technique. Equilibration rates in the London Clay are slow, as evidenced by the retention of suctions in the foreshore in areas first exposed more than 50 years ago. Permeability data from the initial response curves of the piezometers indicates a variation in permeability from $3.5 \times 10^{-10} \text{ m s}^{-1}$ in the weathered upper surface of the clay to $4 \times 10^{-11} \text{ m s}^{-1}$ at 36 m depth. This order of magnitude of permeability change has little effect on the porewater-pressure distribution inland of the slopes at Warden Point. The slow equilibration rate prevents loss of depressed porewater-pressure in the slope since soil is removed faster than moisture penetration takes place.

The cliff-line is retreating at about 1.6 m a^{-1} , taking place as shallow mudslides (Hutchinson, 1970; Hutchinson and Bhandari, 1971) or deep-seated rotational landslides (Hutchinson, 1965, 1973; Bromhead, 1979). The rate of removal is far too rapid for a steady seepage regime to develop in these slopes; instead, the original slightly underdrained, and hence downwards, seepage pattern is modified by the stress-relief-induced porewater effects. Where the slopes recede under the influence of shallow mudslides there is a perched water table in the mudslide debris (Hutchinson and Bhandari, 1971), often with porewater pressures significantly in excess of hydrostatic pressure due to undrained loading. Beneath these, in the in-situ clay, pore-water pressures may be lowered substantially, possibly becoming negative in places. Such depressed porewater-pressures exist only in the vicinity of the cliff-face, as they are carried back inland with retreat of the cliff-face.

Porewater pressure/depth relationships are comparable for the two sections with the exception of the two cliff-edge installations. These are similar down to a depth of about 18 m, after which the porewater pressures are considerably greater for Section 2. The most probable explanation for this is that the existence of the large landslide block 20 m down the slope reduces the lateral stress-relief below 20 m and leaves higher porewater-pressures.

(c) Bromhead and Dixon on slope stability

First-time slides that occur along the north Sheppey coast are of particular interest because, unlike most failures in London Clay, they are located substantially in the unweathered (in-situ) material. This material therefore has a controlling influence on the mechanism of failure, and hence on the shear strength that is mobilized during the first-time sliding.

As shown, the failure mechanisms of the slopes at Warden Point are controlled by the magnitude and distribution of the depressed porewater-pressures. Slide behaviour in these cliffs is dominated by the effects of the stress-relief-induced suctions. If artificially stabilized, the long-term tendency of the porewater pressures to rise will result in destabilization. Using the above measurements Bromhead and Dixon (1986; Dixon and Bromhead, 1991) carried out an investigation based on back-analysis of the slide at K39, to provide information on the field residual shear-strength of London Clay that is mobilized during re-activated slide movements. Unfortunately the data obtained was still insufficient to enable a detailed back-analysis of one of the first-time failures to be carried out and hence to gain accurate information on the distribution of mobilized shear-strength. However, an analysis using a typical 'idealized' pre-failure slope profile, with an idealized porewater-pressure distribution and idealized slip-surface was attempted. The slope profile was based on the 1984 survey of K38, which was considered to be representative of the angle of the cliffs at Warden Point prior to failure. The porewater pressures were those measured on Section 1 at K38, as these related to the chosen slope profile and hence the magnitude of stress-relief experienced during the formation of the slope. A slip-surface with a curved rear section and planar basal portion ((Figure 8.8)d) was considered relevant as discussed above. (Figure 8.11) shows details of the problem analysed.

Using the Morgenstern and Price (1965) slope stability method the initial analysis assumed a uniform distribution of shear strength around the slip-surface. For failure to occur an average shear-strength given by the parameters $c' = 0 \text{ k Nm}^{-2}$ (assumed), $\phi = 16^\circ$ is required. However, there is no field evidence to suggest that it is possible for the shear strength of London Clay to drop to this value in a first-time slide. Skempton (1970) proposed that the parameters $c' = 0$, $\phi = 20^\circ$ give the realistic limit for the drop in strength preceding failure. This conclusion is based on the fact that unrealistically large pre-failure deformations are required to reduce the shear-strength parameters below these values. It appears from this analysis that a non-uniform distribution of shear strength is needed around the slip-surface, and that a major part of the

surface must have substantially reduced strength (Dixon and Bromhead, 1991).

Dixon and Bromhead (1991) carried out an additional analysis with the curved and planar sections of the slip-surface given different strengths. The shear strength mobilized along the curved part should be lower than the peak strength of London Clay, typically taken as $c' = 20 \text{ k Nm}^{-2}$, $\phi' = 20^\circ$, due to the progressive failure that will occur as stability decreases. Progressive failure was a factor in the 1971 slide at K39. This was demonstrated by the observation, made on the rear scarp shortly after failure, that the zone of softened clay contained numerous minor shear surfaces. The parameters $c' = 0$, $\phi' = 20^\circ$ were used as they represented the lowest value of strength which could be sensibly expected. The shear-strength parameters required along the planar section of the slip-surface for limit equilibrium, were back-analysed as $c' = 0$ (assumed), $\phi' = 14^\circ$ (i.e. only a few degrees above the residual value).

Deformations in the order of tens of centimetres are normally needed to substantially reduce the shear strength of intact London Clay. However, along discontinuities the shear strength can be lowered close to the residual value after only small displacements (i.e. in the order of tens of millimetres) as demonstrated by Skempton and Petley (1967). If it is assumed that the basal portion of the slip-surface is a bedding-plane-type feature, only small pre-failure movements would be required to reduce the shear strength to the back-analysed values of $c' = 0$, $\phi' = 14^\circ$ (Dixon and Bromhead, 1991).

Despite the different failure conditions observed in the recent landslide at K38 the general mode of failure in the cliffs around Warden Point can still be classified as rotational sliding. However, considering the information on the individual first-time slides, it can be seen that the position and shape of the slip-surface changes from one section of the cliff to another. The most probable explanation for this is that the base of each slide is controlled by planes of weakness, the heights of which alter along the length of the coast. Field evidence from K37 indicates that the base of the slip-surface follows the dip of the bedding, and may in fact be a bedding plane. This demonstrates that the failure mechanism could be controlled by a bedding-related feature. The north-west dip of the London Clay in this area results in a 0.5° component of the dip parallel to the line of the coast. This means that the depth of a particular bed/plane will increase from east to west.

Field observations indicate that a bedding-related feature in the London Clay controls the depth and shape of the slip-surface. In order for the cliffs at Warden Point to fail at the slope angle observed, it is necessary for part of the slip-surface to obtain shear strength close to the residual value. Dixon and Bromhead (1991) have proposed a mechanism whereby the stress-relief that accompanies formation of the slope leads to deformations along the bedding feature, thus substantially reducing its shear strength prior to general slope failure.

Dixon and Bromhead (1991) point out that the above 'idealized' analysis can only give an indication of the magnitude and distribution of shear strength mobilized during a first-time slide. However, they considered that any inaccuracies in the specified slope profile, porewater pressures or slip-surface geometry, would be of insufficient magnitude to alter the general conclusions.

A mechanism for causing these strength reductions is readily available. The horizontal stress-relief that accompanies formation of the cliffs will result in general horizontal slope movements. The presence of a bedding plane would act as a strain concentrator, thus forming a plane within the slope that has a considerably reduced shear strength (Dixon and Bromhead, 1991).

Moving along the cliffs from K40 to K37, the basal slip-surface of the slides could be controlled by different bedding-related planes of weakness. This would explain the apparent change in failure mode from a deep-seated relatively circular slip at K40, through a non-circular deep-seated slide at K39, to a non-circular toe slip at K37. It is also worth noting that a planar sub-horizontal shear surface, which crops out at foreshore level, would help to account for the translational movements observed during the toppling type failure at K38.

It is of interest that the available histories on first-time slides in over-consolidated clays (see for example Barton, 1984) show that the large majority have the basal part of their slip-surface controlled by bedding planes. It is not unrealistic to suggest that this is also the case for slides in the unweathered London Clay. This idea is very close to the emerging idea

of pre-failure displacement under the load of a cap-rock, as, for example, at Castle Hill, Kent, and at the famous landslide 'Le Chaos' on the Bessin Cliffs in Normandy, where the 'zone de decompression' is used as a fundamental planning boundary (Maquaire and Gigot, 1988).

Conclusions

Owing to the stabilization measures taken at other similar sites (at and near Herne Bay, Kent, and Walton, Essex) Warden Point is the only site where the cyclic mechanism described by Hutchinson (1973) is still in operation. Areas of cliff representing all the stages except Stage 5 can currently be seen there. Warden Point represents the condition of London Clay cliffs subject to active marine erosion, although there is disagreement about the significance of this. At Warden Point the porewater pressures do not equilibrate under the ambient hydraulic boundary conditions. Instead, they are held in dynamic equilibrium by the rapid retreat of the cliff-line.

Considering the recent landslides that have occurred in the cliffs at Warden Point, it can be seen that although their failure mechanisms are in most cases essentially the same, there are differences in the shape and position of the slip-surface which can be deduced from field observations. Assessing the landslides in turn, moving from east to west (K40 to K37) it becomes apparent that a change occurs in the mechanism of the first-time failures.

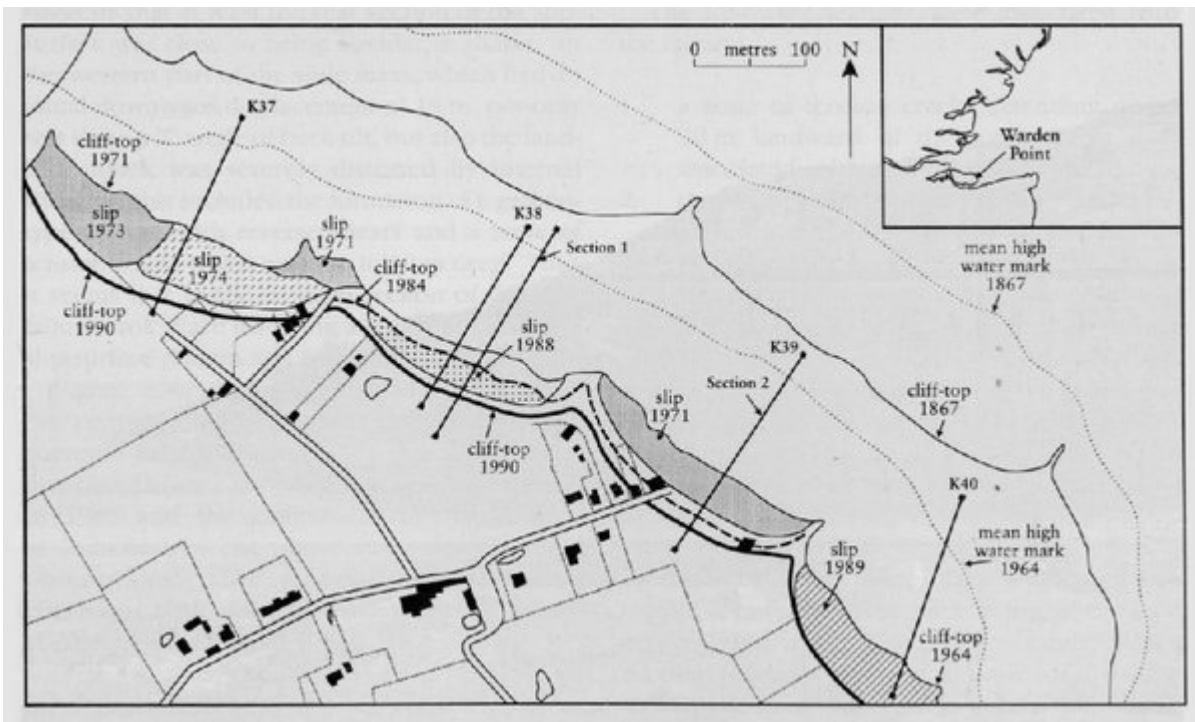
First-time slides occur in slopes that have been steepened by marine erosion. Slide behaviour is controlled by the stress-relief-modified porewater regime. Assessment of the field-mobilized shear strength and the failure mechanism require a detailed understanding of these depressed porewater-pressures.

The toppling mode of failure that occurred at K38 in 1988, while of considerable interest, is not common on slopes in London Clay. Further work is required to assess the special conditions that existed for this type of failure mechanism to take place preferentially to rotational sliding.

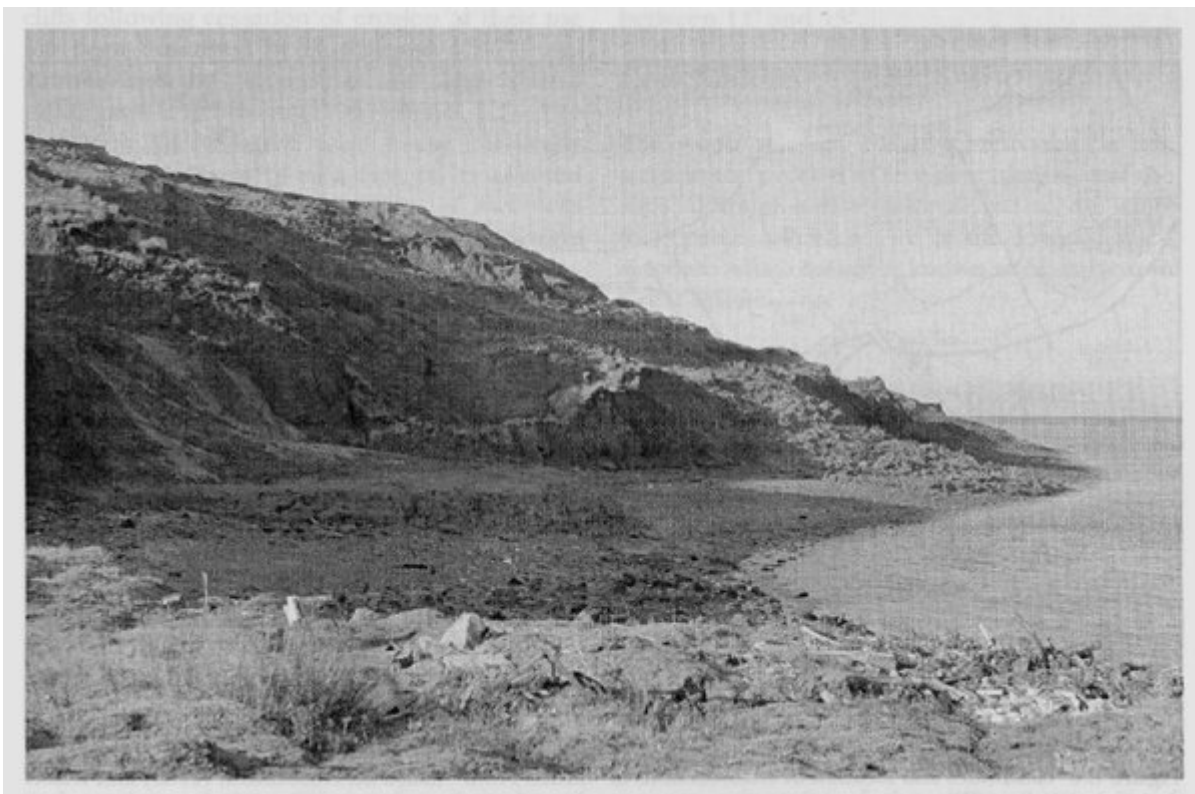
References



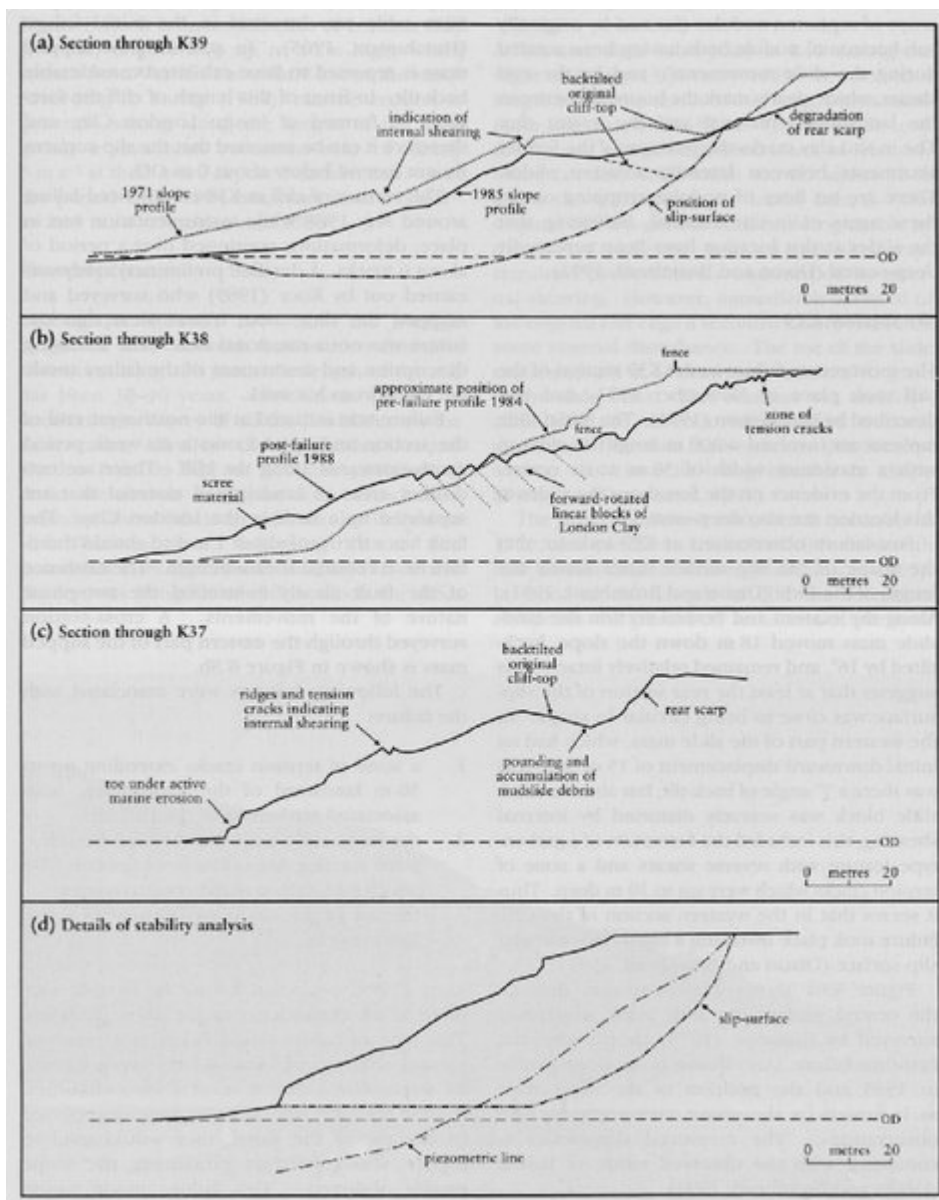
(Figure 8.5) The characteristic morphology of the Warden Point GCR site. (Photo: R.G. Cooper.)



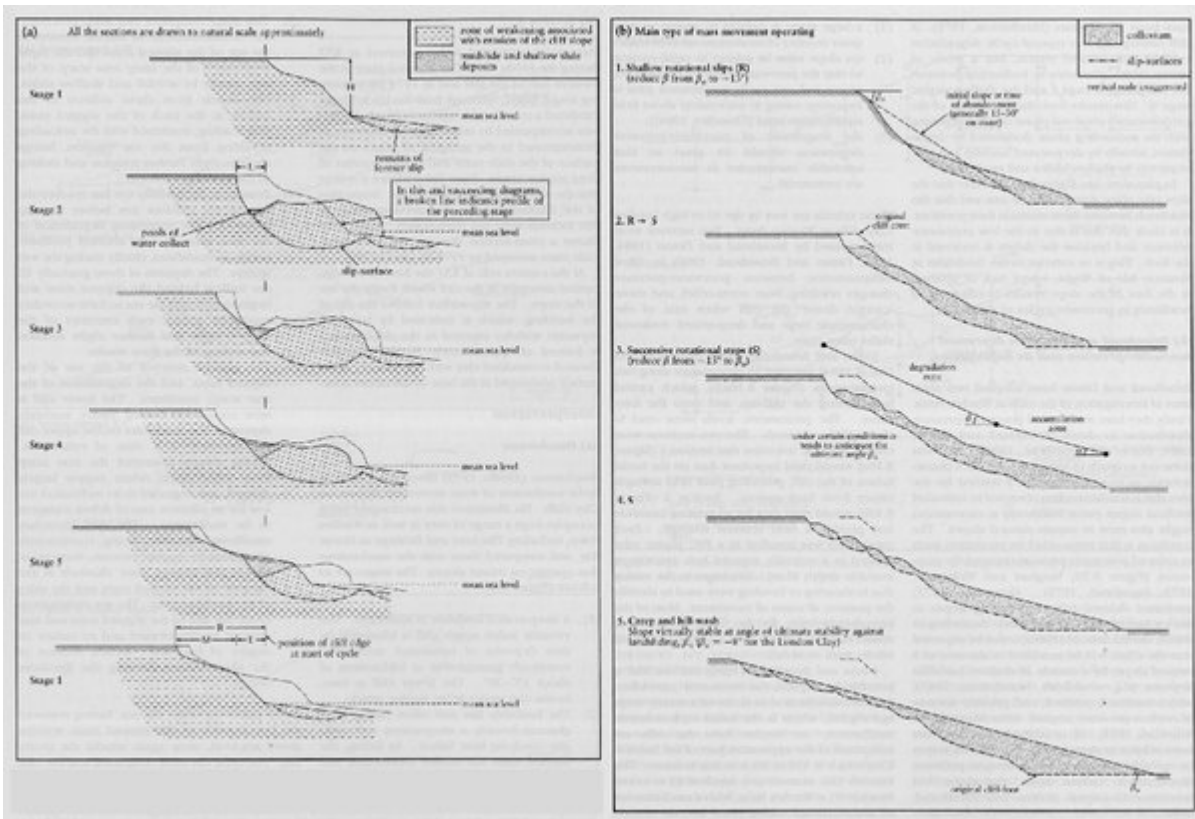
(Figure 8.6) Landslides and location of cross-sections at Warden Point. Based on Hutchinson (1965) and Dixon and Bromhead (1991).



(Figure 8.7) The cliffs at Warden Point looking westwards. Separate mudslide tongues are clearly shown. (Photo: J. Larwood, English Nature/Natural England.)



(Figure 8.8) Sections K37–9 (a–c) at Warden Point (after Lee, 1976; Koor, 1989; Dixon and Bromhead, 1991), and (d) K38 'idealized' profile and slip-surface (after Bromhead and Dixon, 1984). See (Figure 8.6) for locations of sections.



(Figure 8.9) (a) Cyclic mechanism of mass movements in London Clay Cliffs, North Kent. After Hutchinson (1967, 1970, 1973). (b) Cyclic mechanism of mass movements in London Clay Cliffs, North Kent; for comparison with with (Figure 8.9)a. After Hutchinson (1967, 1970, 1973).

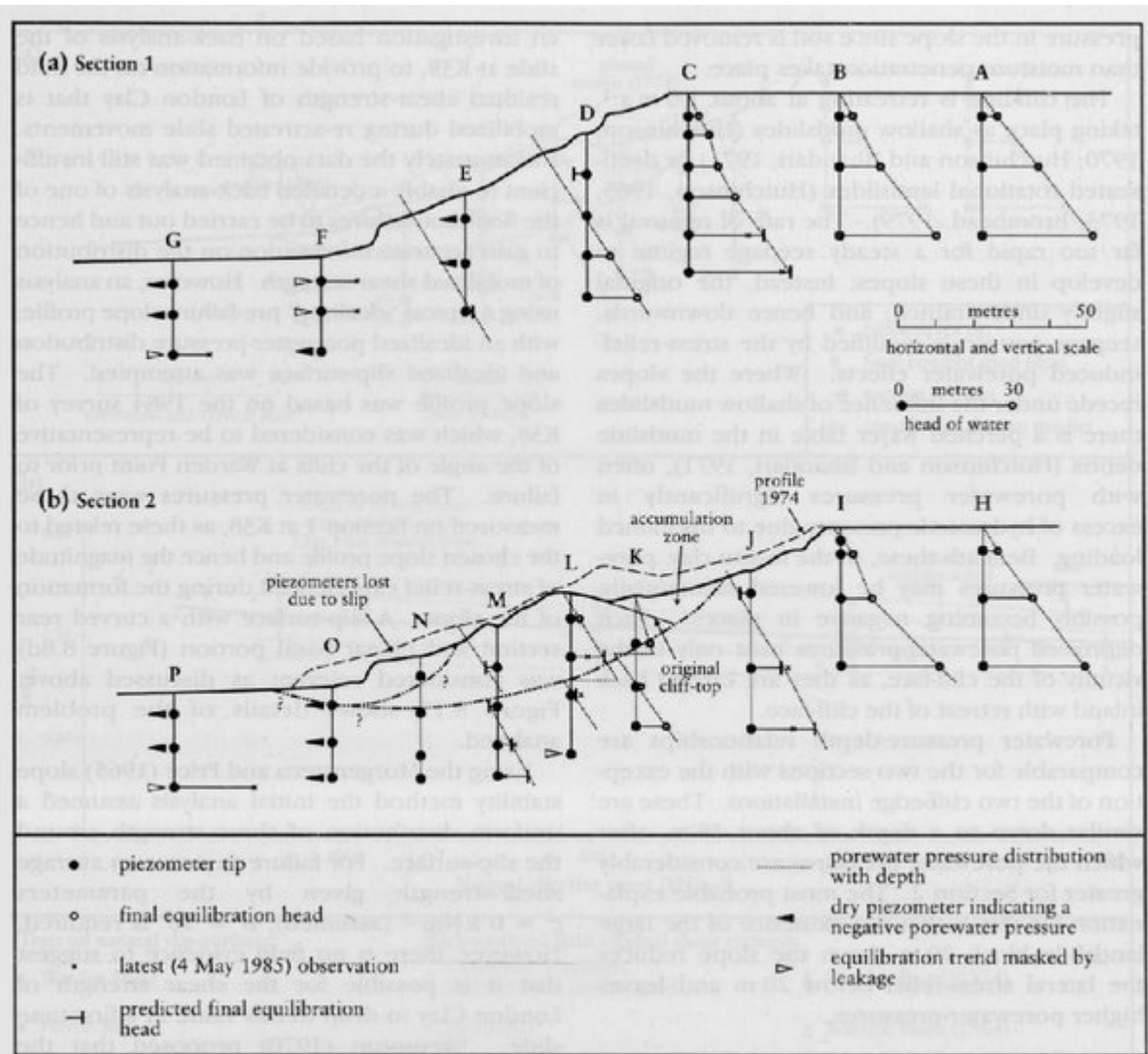
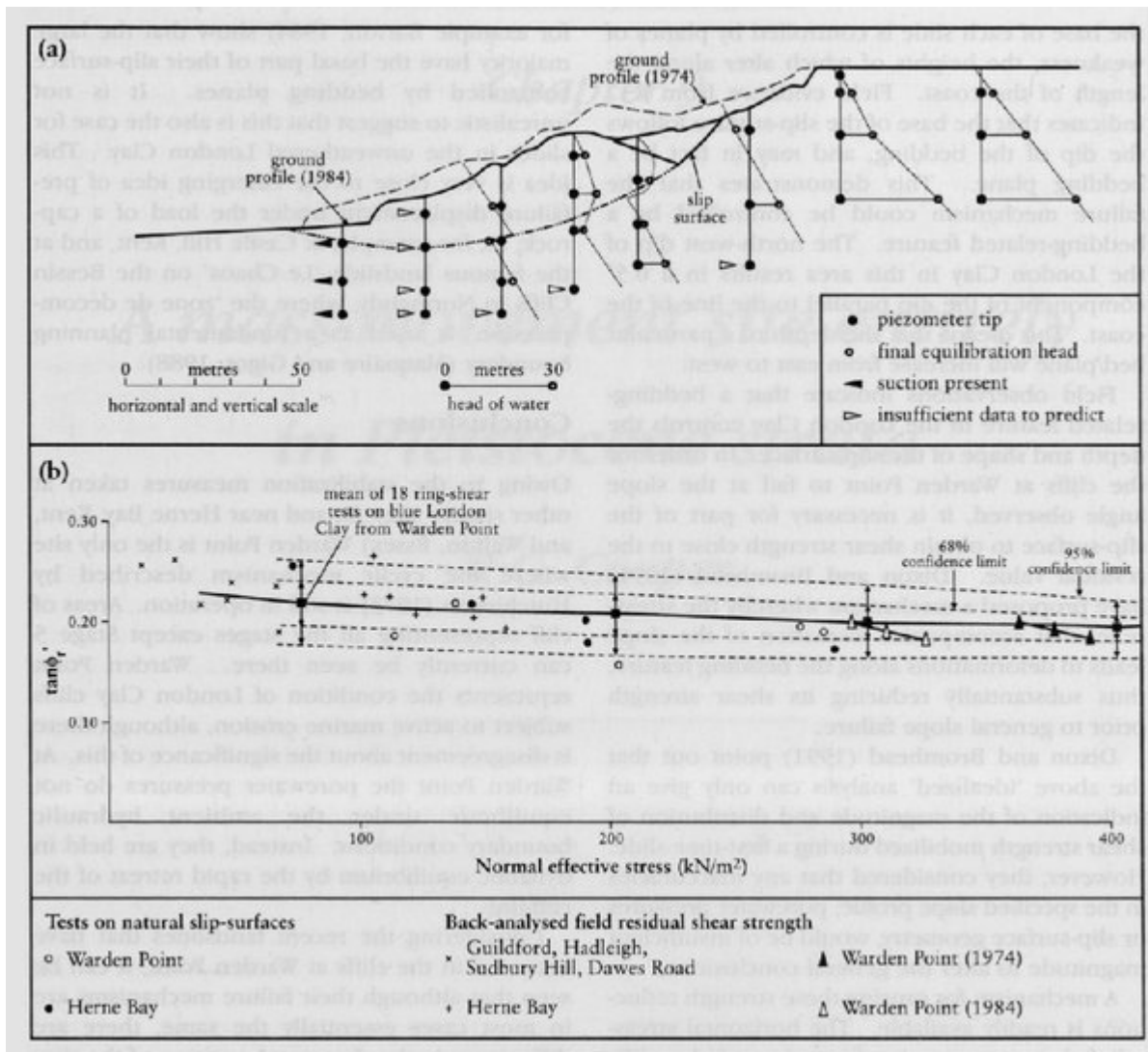


Figure 8.10 Porewater pressure distributions at two sections at Warden Point: (a) pre-failure; and (b) post-failure. After Bromhead and Dixon (1984).



(Figure 8.11) Results of back-analysis of the 1984 landslide at section K38, Warden Point. Based on Bromheaci and Dixon (1984) and Dixon and Bromhead (1991.)